### Proof by Contradiction

Sometimes it is B difficultB to find direct proof of the form A A → BA for an assertion B B. Some-

times it can be easier to prove the statement by contradicting it. This is also referred to as an indirect proof. To do this, proceed as follows: Condition applies and it is assumed that is wrong. In the proof one should show that the assumption that is wrong cannot

ment B. be true. It follows that must be correct. Proof by contradiction is based on the laws of logic. They rely on the fact that a true statement can never be followed by a false state• Let n ∈ ℕ be an even natural number with n n ∈ ℕ. Then n is also an even number. Example: Proof by contradiction

Proof:

Let’s assume would be odd. We have seen in the previous example of direct proofs that the square of odd numbers is also odd. It would therefore follow thatn2 = n n

• Proof:Let a, b ∈ ℝ+. Then the following applies:a+b2 ≥ ab □ quently, the assumption n would be odd is wrong. is also odd. But this is a contradiction to the condition that is an even number. Conse-

Assuming thatthen it follows that a+b2 < ab

+2ab

is wrong. □ a+b2 < ab